

## SM3 8.1 Exponential and Log Equations

Because exponential and log are inverse operations, we can use them to undo one another while solving equations.

### Solving exponential equations

Step 1) Isolate the exponential term, if necessary.

Step 2)  $\log_b(\ )$  both sides of the equation, where  $b$  is the base of the exponential.

Step 3) Let log cancel exponential. Evaluate remaining logs.

Step 4) Conclude solving using algebra skills.

Step 5) Check for accuracy, if time permits. Because the domain of exponentials is all real numbers, there is no need to check your answers for extraneous solutions.

Example: Solve  $5^{x-4} = 125$

$\log_5(5^{x-4}) = \log_5(125)$	$\log_5(\ )$ both sides of the equation
$x - 4 = 3$	Cancel left log/exp; evaluate right log
$x = 7$	Addition

Example: Solve  $3^{x+6} = 81$

Example: Solve  $2^{x-7} + \frac{1}{4} = \frac{3}{4}$

### Solving log equations

Step 1) Isolate the log term, if necessary. This might require log properties. It's acceptable to have a single log on each side of the equation.

Step 2)  $b(\ )$  both sides of the equation, where  $b$  is the base of the log.

Step 3) Let exponential cancel log. Evaluate remaining exponentials.

Step 4) Conclude solving using algebra skills.

Step 5) Because the domain of logs is limited, check for extraneous solutions! When you plug the solution back into the log, it must not result in the log evaluating zero or a negative number.

Example: Solve  $\log_5(x-2) = 3$

$5^{\log_5(x-2)} = 5^3$	$5(\ )$ both sides of the equation
$(x-2) = 125$	Cancel left log/exp; evaluate right expo
$x = 127$	Addition

Does  $x = 127$  make a log evaluate zero or a negative? No, the log evaluates 125. Keep  $x = 127$

Example: Solve  $\log_7(2x - 1) = \log_7(x + 4)$

$7^{\quad}$  both sides of the equation  
Cancel left log/exp  
Addition

Does  $\quad$  make a log evaluate zero or a negative?

Example: Solve  $\log_2 x + \log_2(x + 4) = 5$

Log property

### HW 8.1

Solve the following equations.

1.  $3^{n-2} = 27$

2.  $2^{3x+5} = 128$

3.  $5^{n-3} = \frac{1}{25}$

4.  $10^{x-1} = 100^{2x-3}$

5.  $\log_9 x = 2$

6.  $\log_{25} n = \frac{3}{2}$

7.  $\log_{\frac{1}{7}} x = -1$

8.  $\log(x^2 + 1) = 1$

9.  $\log_b 64 = 3$

10.  $\log_5 5^{6n+1} = 13$

11.  $\log_5 x = \frac{1}{2}$

12.  $\log_b 121 = 2$

13.  $\log_6(2x - 3) = \log_6(x + 2)$

14.  $\log_7(x^2 + 36) = \log_7 100$

15.  $\log_3 5 + \log_3 x = \log_3 10$

16.  $\log_4 a + \log_4 9 = \log_4 27$

17.  $\log 16 - \log 2t = \log 2$

18.  $\log_7 24 - \log_7(y + 5) = \log_7 8$

$$19. \log_2 n = \frac{1}{4} \log_2 16 + \frac{1}{2} \log_2 49$$

$$20. 2 \log 6 - \frac{1}{3} \log 27 = \log x$$

$$21. \log z + \log(z + 3) = 1$$

$$22. \log_6(a^2 + 2) + \log_6 2 = 2$$

$$23. \log_2(12b - 21) - \log_2(b^2 - 3) = 2$$

$$24. \log_2(y + 2) - \log_2(y - 2) = 1$$

$$25. \log_3 0.1 + 2 \log_3 x = \log_3 2 + \log_3 5$$

$$26. \log_5 64 - \log_5 \frac{8}{3} + \log_5 2 = \log_5 4p$$

$$27. 2e^x - 1 = 0$$

$$28. -3e^{4x} + 11 = 2$$

$$29. \ln 2x = 4$$

$$30. \ln 3x = 5$$

$$31. \ln(x + 1) = 1$$

$$32. \ln(x - 7) = 2$$

$$33. \ln x + \ln 3x = 12$$

$$34. \ln 4x + \ln x = 9$$

$$35. \ln(x^2 + 12) = \ln x + \ln 8$$

$$36. \ln x + \ln(x + 4) = \ln 5$$

$$37. e^{\ln x} = 4$$

$$38. 200e^{-4x} = 15$$

$$39. \ln x^2 = 10$$

$$40. 9 - 2e^x = 7$$

$$41. \ln \sqrt{x + 2} = 1$$

$$42. \ln(x - 2)^2 = 12$$